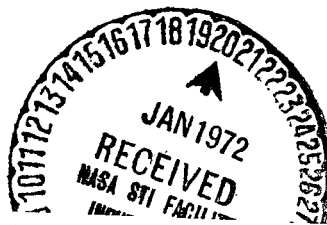NASA TECHNICAL TRANSLATION

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ELECTRON-OPTICAL CYLINDER LENS ACTION OF THE STRAY FIELDS
OF A CONDENSER

Richard Herzog

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ELECTRON OPTICAL CYLINDER LENS ACTION OF THE STRAY FIELDS
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R. Herzog

ABSTRACT: Deflection in the stray field of a condenser where the field is limited by a stop is computed and compared with that of a homogeneous substitution field. In both cases the axis beam is sharply deflected to the same degree if the substitution field overlaps the condenser plates by the distance ξ; it can be read directly from Figure 3 for any stop position. The length of the substitution field can also be taken from Figure 3 for screened magnetic fields, providing that the magnetization of the iron is sufficiently short of saturation.
Rays which do not pass the stray field in the axis are deflected differently from those in the substitution field. Since this additional angle of deflection $\Delta\alpha$ is proportional to the distance from the axis, the activity of the stray field can be described through a thin lens, the focal distance of which can be taken from Figures 4 and 5 and from equation (7). However, for most practical cases the focal distance is so large that it may be disregarded. This indicates that, in computing the lens action of transgradient electrical fields instead of the real field, using a sharply defined substitution field for the computation is justified.

I. Introduction

The deflection of a beam of charged particles in a condenser has usually /18* been computed by assuming that the field outside the space defined by the plates equals zero and is homogeneous inside it. However, in actual fact the field changes continuously from zero to its maximum value along a line which is comparable to the plate interval. The purpose of this work is to find out how large an error is made in approximating the real field shape by a sharply defined homogeneous substitution field. The result is anticipated: the field of a condenser can be described very closely by a homogeneous substitution field, but the length of the latter does not generally agree with the length of the /19 condenser plates; how this substitution length is to be figured will be shown

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*Numbers in the margin indicate pagination in the foreign text.

during the article. For exact research an additional cylinder lens action of the stray fields must be considered, and this overlaps the cylinder lens action of the substitution field. In computing the lens properties of an ideal cylinder condenser [1] or of an ideal plane plate condenser [2] with sharply defined fields, consideration of the difference in potential at the edge of the field was essential and in the final case this was the unique cause for the lens properties. Now, in order to observe the action of the stray fields, we shall compute the paths of the particles in the real condenser and compare them with those in the substitution field with a difference in potential.

2. Deflection of the Beams at the Edge of the Field

Figure 1 shows the position of the coordinate system in reference to the condenser plates and stops. The homogeneous substitution field begins at position $x = d/2 - \xi$, i.e., the substitution field protrudes over the condenser plates by the distance ξ. We wish to compute ξ in such a way that a beam entering the condenser on the x axis will be deflected just as strongly as in the substitution field.
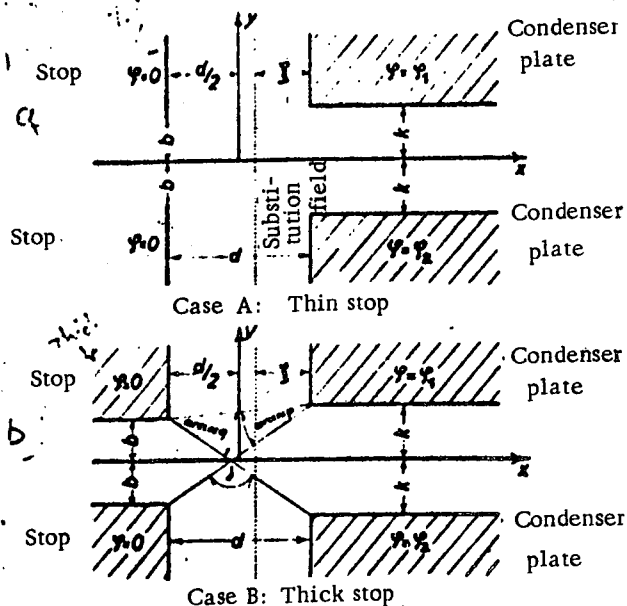
V designates the volt velocity of the particles far before the stop where the potential is $\phi = 0$; let V be positive (negative) for particles with the charge +e (-e). At a position with potential ϕ, the velocity v of the particles is provided by $v = \sqrt{(2e/m)(V - \phi)}$. We presume that the beam in the stray field is only slightly deflected; therefore we can substitute the field intensity and the potential in the real path by the values of those magnitudes on the linear

Figure 1. Position of the Coordinate Systems and Designation of the Dimensions.

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extension of the path of the entering beam. In addition we presume that the angle of deflection α and ϕ/V are small magnitudes whose squares can be disregarded. With these omissions we get

$$\alpha = \frac{I + \frac{\phi_i}{2V}}{2V} \cdot \int_{-\infty}^{\tau} \left(I + \frac{\phi}{2V} \right) F_y dx. \qquad (1)$$

Here ϕ means the potential at the field position where the beam is deflected by α.

In the homogeneous substitution field

$$\phi = \frac{\phi_1 + \phi_2}{2} + \frac{y}{k} \cdot \frac{\phi_1 - \phi_2}{2}$$

for

$$F_y = \frac{\phi_2 - \phi_1}{2k}.$$

and $\phi = 0$ and $F_y = 0$ for $x < d/2 - \xi$

If we introduce this into equation (1), we get

$$\alpha_H = \frac{I + \phi_i/2V}{2V} \cdot \frac{\phi_2 - \phi_1}{2k} \cdot \left(I + \frac{\frac{\phi_1 + \phi_2}{2} + \frac{y}{k} \frac{\phi_1 - \phi_2}{2}}{2V} \right) \left(x - \frac{d}{2} + \xi \right) \qquad (2)$$

3. Deflection in the Real Stray Field

The potential and the field intensity in the vicinity of the axis can be computed from the potential and the field intensity on the axis by means of equations

$$\phi = \phi_0 - y F_{0_y} \quad \text{and} \quad F_y = F_{0_y} - y \frac{d}{dx} F_{0_y}.$$

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In these the index 0 signifies the value on the axis (y = 0); below, for the sake of simplicity, we shall disregard the index 0. If the above equations are inserted into equations (1), if the quadratic members in y are disregarded, if equation (2) is subtracted from it and extended infinitely to the boundary x, we get the additional deflection caused by the stray field:

$$\Delta\alpha = \alpha - \alpha_{H} = \frac{I + \frac{\phi_i}{2V}}{2V} \cdot \lim_{x \to \infty} \left[\int_{-\infty}^{x} \left(I + \frac{\phi}{2V} \right) F_y dx - \left(I + \frac{\phi_1 + \phi_2}{4V} \right) \frac{\phi_2 - \phi_1}{2k} \right.$$

$$\left. \left(x = \frac{d}{2} + \xi \right) \right] + \frac{y}{4V^2} \cdot \lim_{x \to \infty} \left[\left(\frac{\phi_2 - \phi_1}{2k} \right)^2 \left(x - \frac{d}{2} + \xi \right) - \int_{-\infty}^{x} \left(F_x^2 + F_y^2 \right) dx \right] \tag{3}$$

This infinite extension seems to be in contradiction with the condition of a small deflection. Supposedly the integration should only be extended to that value of x for which the field is already practically homogeneous, but the deflection is still small; as long as the radius of curvature of the path in the field is large when compared with the plate interval, there exists a position x which fulfills the above conditions. However, since the integrant has already assumed the value of the homogeneous field for larger values of x, nothing is then changed in the difference $\Delta\alpha$ if the integration is extended infinitely.

If this first limiting value disappears, it means that the axis beam y = 0 is deflected with equal sharpness in both the real and the substitution condenser. The condition for this is:

$$\xi = \lim_{x \to \infty} \left[\frac{d}{2} - x + \frac{I}{\left(I + \frac{\phi_1 + \phi_2}{4V} \right) \frac{\phi_2 - \phi_1}{2k}} \cdot \int_{-\infty}^{x} \left(I + \frac{\phi}{2V} \right) F_y dx \right] . \tag{4}$$

The portion of equation (3) multiplied by y shows that from a parallel pencil of rays there comes one beam with a focal point at position x = f = -y/$\Delta\alpha$. We find:

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$$\frac{I}{f} = + \frac{I}{4V^2} \cdot \lim_{x \to \infty} \left\{ \int_{-\infty}^{x} F_y^2 dx - \left(\frac{\phi_2 - \phi_1}{2k} \right)^2 \left(x - \frac{d}{2} + \xi \right) \right\} + \frac{I}{4V^2} \int_{-\infty}^{+\infty} F_x^2 \, dx \; . \quad (5)$$

In order to be able to evaluate the above integrals, we need the potential and the field intensity in the condenser stray field. These magnitudes were computed in a previous work [3]; let us briefly review the results here. Two extreme cases, as shown in Figure 1, were considered: case A with a very thin stop and case B with a very thick stop; all possible cases lie between these limits.

$$\phi = \frac{\phi_1 + \phi_2}{2\pi} \text{ arc cos } \frac{n - I - 2r}{n + I} ; \qquad (24A)$$

$$\phi = \frac{\phi_1 + \phi_2}{2\pi} \text{ arc cos } \frac{q - p - 2r}{p + q} ; \qquad (24B)$$

$$F_x = - \frac{\phi_1 + \phi_2}{2d} \cdot \frac{I + r}{m + r} \cdot \sqrt{\frac{n - r}{I - r}} ; \qquad (25A)$$

$$F_x = - \frac{\phi_1 + \phi_2}{2d} \cdot \sqrt{\frac{(p + r)(q - r)}{I - r^2}} \qquad (25B)$$

$$F_y = - \frac{\phi_1 - \phi_2}{2d} \cdot \frac{I}{m + r} \cdot \sqrt{\frac{(I + r)^3}{I - r}} ; \qquad (26A)$$

$$F_y = - \frac{\phi_1 - \phi_2}{2d} \cdot \frac{p + r}{\sqrt{I - r^2}} \; . \qquad (26B)$$

Here m, n, p, q are auxiliary magnitudes which depend on the geometric position of the stop. It is best to derive m and n from the adjacent diagram (Figure 2), as well as p and q of a scalar stop diagram according to Figure 1B. Auxiliary

modifying r depends on x by means of equations:

$$x = \frac{d}{\pi} \text{ arc sin } r + \frac{k}{\pi} \mathfrak{Ar}\mathfrak{Cof} \frac{I - nr}{n - r} - 2 \frac{k}{\pi} \cdot \frac{m - I}{m + n} \cdot \sqrt{\frac{(I + n)(I - r)}{(I - n)(I + r)}} \; ; \qquad (22A)$$

$$x = \frac{d}{\pi} \text{ arc sin } r + \frac{k}{\pi} \mathfrak{Ar}\mathfrak{Cof} \frac{I - qr}{q - r} - \frac{b}{\pi} \mathfrak{Ar}\mathfrak{Cof} \frac{I + pr}{p + r} \; . \qquad (22B)$$
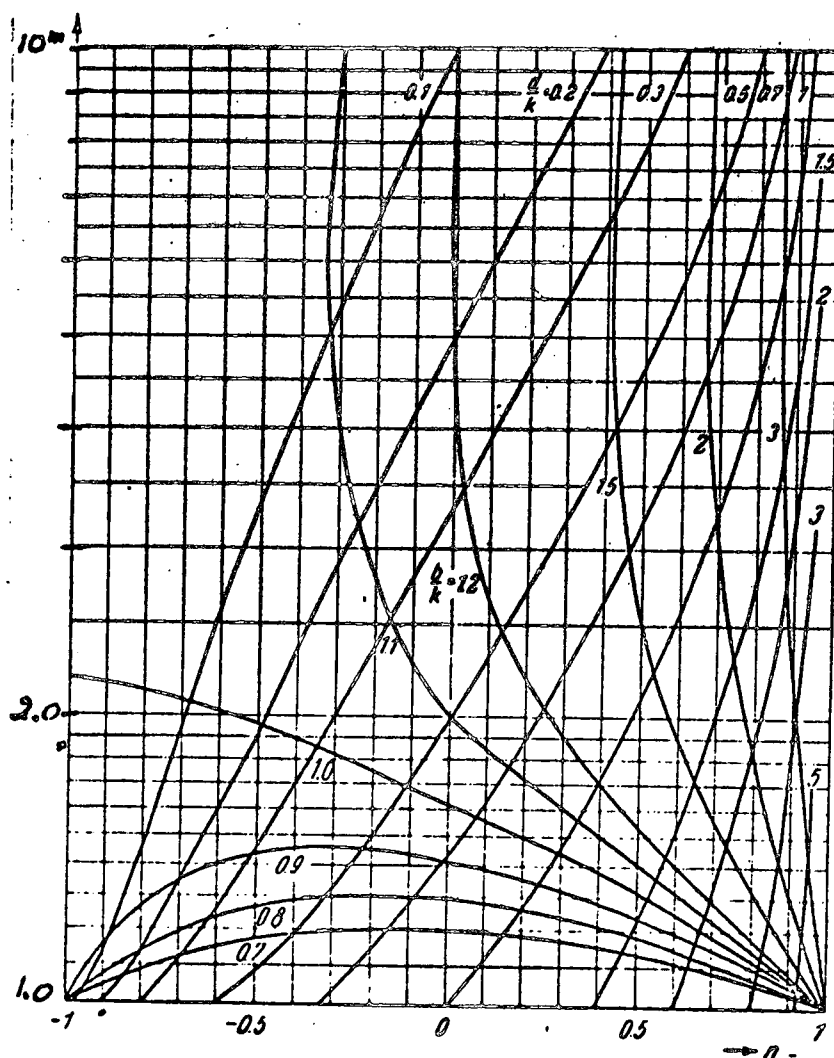

Figure 2. For Deriving the Auxiliary Constants in Case A.

For large values of x (r → n or r → q) these equations are transposed into:

$$x = \frac{d}{\pi} \text{ arc sin } n + \frac{k}{\pi} \ln \frac{2(I - n^2)}{n - r} - \frac{2k}{\pi} \frac{m - I}{m + n} \; ;$$

$$x = \frac{d}{\pi} \text{ arc sin } q + \frac{k}{\pi} \ln \frac{2(I - q)}{q - r} - \frac{b}{\pi} \text{ArCof} \frac{I + pq}{p + q} \; .$$

If equations [22] are differentiated, we get:

$$dx = \frac{d}{\pi} \cdot \frac{m + r}{n - r} \cdot \sqrt{\frac{I - r}{(I + r)^3}} \; dr, \qquad dx = \frac{d}{\pi} \cdot \frac{\sqrt{I - r^2}}{(p + r)(q - r)} \; dr.$$

With the assistance of these references, the integrals in equations (4) and (5) can all be computed. After a rather long intermediate computation, insertion into equation (4) leads to:

$$\frac{\xi}{k} = \frac{I}{\pi} \cdot \left\{ \frac{d}{k} \text{ arc cos } n + 2 \frac{m - I}{m + n} - \ln 2(I - n) - \frac{\phi_1 + \phi_2}{2V} \ln 2 \right\} \; ; \qquad (6A)$$

$$\frac{\xi}{k} = \frac{I}{n} \cdot \left\{ \frac{d}{k} \text{ arc cos } q + \frac{b}{k} \text{ArCof} \frac{I + pq}{P + q} - \ln 2 \frac{I - q^2}{p + q} - \frac{\phi_1 + \phi_2}{2V} \ln 2 \right\} . \qquad (6B)$$

These equations were already derived in an earlier work [4] for the special case of $\phi_1 = -\phi_2$. It was already shown at that time that ξ can be made equal to 0 by a suitable choice of stop distance and interval, i.e., that the ideal substitution field possesses the same length as the condenser plates. This is particularly advantageous with a radial electrical field, because then the equipment axes before and behind the field are perpendicular to the radial plane in which the condenser arcs end.

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Evaluation of equations (6) represent a very extensive piece of computation. In order to make this bearable for the experimenter, ξ/k was computed for a large number of stop positions and the results summarized in Figure 3. The relative stop interval d/k and the relative stop distance b/k were plotted on the coordinate axes; now one point in the diagram corresponds to each stop position; this is simultaneously the picture of the stop edge in a scalar diagram in which a condenser plate has the position indicated and the second plate lies symmetrically on the abscissa. The set of curves connect points with the same ξ/k, where the dark curves refer to a very thick stop and the thin curves to a very thin stop. The transconductance of the field can be disregarded in the case of a small stop distance; for this reason thick and thin stops here have the same action and the corresponding sets of curves touch each other. It is also clear that both the stray field and thus ξ increase with an increase in the stop interval or with an increase in the stop distance. However, the behavior with thin and distant stops ($b > k$) is striking: if such a stop is removed from the condenser plates, ξ first decreases and only increases again at a greater distance. The cause of this lies in the fact that the stop at a short interval screens the stray field on the axis more severely than when it is immediately in front of the plates. Only with a larger stop interval is the field of the front sides of the condenser plates perceptible as far as the axis and the deflection again enlarged.

With an unsymmetrical grounding of the condenser voltage, a short circuit occurs in the homogeneous substitution field by the usually very small amount $\Delta\xi/k = -0.11 (\phi_1 + \phi_2)/V$; it is worth noting that this short is completely independent of the stop. Thus a suitable choice of grounding makes it possible to vary the actual condenser length within narrow limits and thus to compare small errors in the mechanical production. The influence of the grounding (computed here) on the length of the substitution field works against this deflection error [5] and, the greater the plate interval compared to the length, the more it reduces the deflection error.
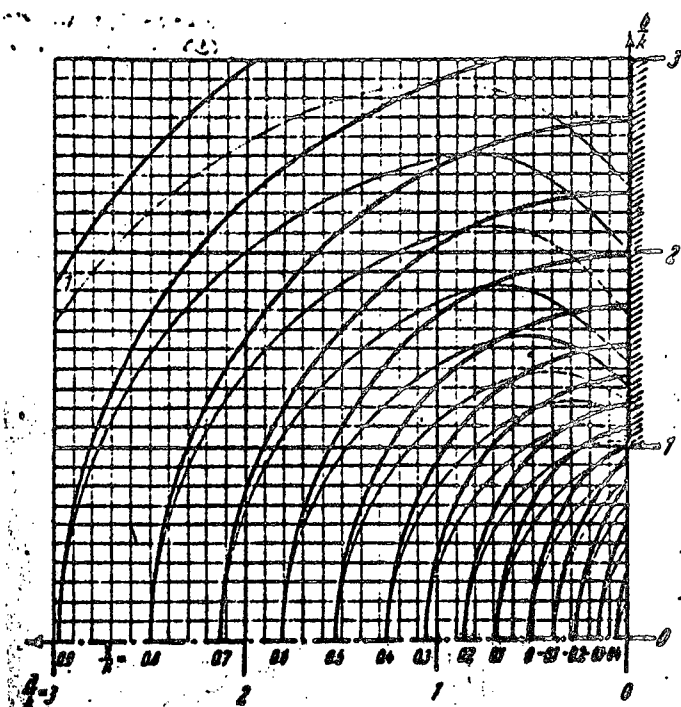
Figure 3. The Substitution Field Overshadows the Condenser Plates by the Distance ξ. The thin curves refer to a thin stop and the thick curves to a thick stop. The hatched corner represents the beginning of a condenser plate which is extended infinitely at the right top; the second plate is symmetrical to the d/k axis.

The lens operation of the stray field is obtained by substitution in equation (5); after a rather long intermediate computation during which the equations

$$\frac{d}{k} = \frac{I}{m + n} \cdot \sqrt{\frac{(n + I)^3}{I - n}} \quad \ldots \quad (4\Lambda)$$

and

$$\frac{d}{k} = \frac{p + q}{\sqrt{I - q^2}} \quad \ldots\ldots \quad (4B)$$

must be considered, we get

$$\frac{I}{f} = \frac{I}{4V^2} \cdot \lim_{\substack{x \to \infty \\ r \to n}} \left\{ \frac{(\phi_2 - \phi_1)^2}{4\pi d} \left[\frac{(m - I)^2}{(m + n)\sqrt{m^2 - I}} \text{ arc cos} \right. \right.$$

$$\left. \left(-\frac{I + mr}{m + r} \right) + \frac{d}{k} \, \mathfrak{Ar}\mathfrak{Cof} \, \frac{I - nr}{n - r} - \text{arc cos}(-r) \right] - \left(\frac{\phi_2 - \phi_1}{2k} \right)^2 \cdot \frac{k}{\pi} \left[\ln \frac{n + I}{n - r} - \right.$$

$$\left. - \frac{\phi_1 + \phi_2}{2V} \ln 2 \right\} + \frac{I}{4V^2} \cdot \frac{(\phi_1 + \phi_2)^2}{4\pi d} \left[\text{arc cos } (-n) - \sqrt{\frac{m - I}{m + I}} \text{ arc cos} \right.$$

$$\left. \left(-\frac{I + mn}{m + n} \right) \right] \right]. \qquad (A)$$

$$\frac{I}{f} = \frac{I}{4V^2} \cdot \lim_{\substack{x \to \infty \\ r \to q}} \left\{ \frac{(\phi_2 - \phi_1)^2}{4\pi d} \left[\frac{d}{k}\,\mathfrak{Ar\,Cof}\,\frac{I - qr}{q - r} - \frac{d}{k}\,\mathfrak{Ar\,Cof}\,\frac{I + pq}{p + q} - \arccos(-r) \right. \right.$$

$$\left. + \arccos p \right] - \left(\frac{\phi_2 - \phi_1}{2k}\right)^2 \frac{k}{\pi} \left[\ln \frac{p+q}{q-r} - \frac{\phi_1 + \phi_2}{2V}\,\ln 2 \right] \Bigg\} \qquad \text{(B)}$$

$$+ \frac{I}{4V^2}\,\frac{(\phi_1 + \phi_2)^2}{4\pi d}\,[\arcsin p + \arcsin q].$$

We now split f according to

$$\frac{I}{f} = \frac{I}{f_1} + \frac{I}{f_2} \qquad (7)$$

into one portion f_1, which depends only on the field intensity and a portion f_2 which also depends upon the type of grounding of the plates and is infinite with symmetrical current. If we disregard all members which contain V^3 in the denominator in these equations and put on the right side only the members dependent upon the field's shape, we get

$$\frac{f_1}{k}\left(\frac{\varphi_1 - \varphi_2}{4V}\right)^2 = \frac{\pi}{\frac{k}{d} \cdot \frac{I}{m+n} \cdot \sqrt{\frac{(m-I)^2}{m+I}}\,\arccos\left(-\frac{I+mn}{m+n}\right) - \frac{k}{d}\arccos(-n) + \ln 2(I-n)} \; ; \quad \text{(8A)}$$

$$\frac{f_1}{k}\left(\frac{\varphi_1 - \varphi_2}{4V}\right)^2 = \frac{\pi}{\frac{k}{d} \cdot \arccos p - \frac{k}{d}\arccos(-q) - \mathfrak{Ar\,Cof}\,\frac{I+pq}{p+q} + \ln 2\,\frac{I-q^2}{p+q}} \; ; \quad \text{(8B)}$$

$$\frac{f_2}{k}\left(\frac{\varphi_1 + \varphi_2}{4V}\right)^2 = \frac{\pi}{\frac{k}{d}\arccos(-n) - \frac{k}{d}\sqrt{\frac{m-I}{m+I}}\,\arccos\left(-\frac{I+mn}{m+n}\right)} \; ; \quad \text{(9A)}$$

$$\frac{f_2}{k}\left(\frac{\varphi_1 + \varphi_2}{4V}\right)^2 = \frac{\pi}{\frac{k}{d}\arcsin p + \frac{k}{d}\arcsin q} = \frac{\pi \cdot d}{\gamma \cdot k} \; . \quad \text{(9B)}$$

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Figure 1B provides the magnitude of y.

In order to facilitate practical use the above equations were also worked out for large number of stop positions and the results summarized in Figures 4 and 5. From them it is possible to read directly and quite precisely the right side of equations (8) and (9) dependent upon the stop position, especially if the lens action of the stray fields is small compared with that of the main field.
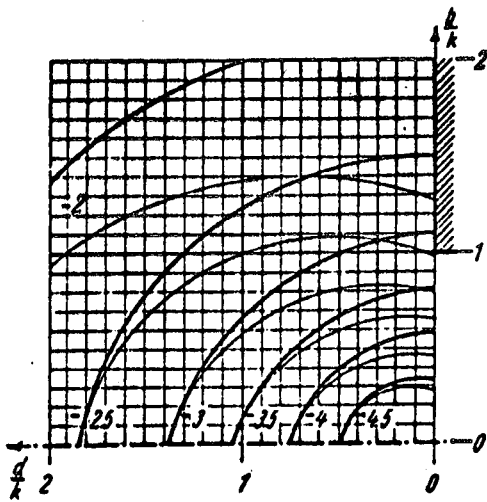

Figure 4. The Parameter Signifies
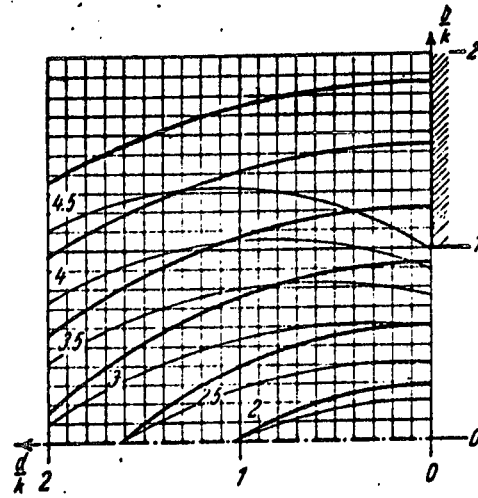
$$\frac{f_1}{K} \quad \frac{\phi_1 - \phi_2}{4V}^2 \quad .$$

Figure 5. The Parameter Signifies

$$\frac{f_2}{K} \quad \frac{\phi_1 + \phi_2}{4V}^2 \quad .$$

4. Lens Activity of the Entire Field

The deflection which a charged particle experiences in going through a real field is composed linearly of the deflections in the two stray fields and in the homogeneous substitution field. Therefore the lens operation of the entire field can be obtained by combining the lenses of the stray fields and of the substitution field, as in geometrical optics. The corresponding equations for three fields are [6]:

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$$\xi_1 = \frac{f_1 f_1' \Delta_{2,3}}{\Delta_{1,2} \Delta_{2,3} + f_2 f_2'};$$

$$\xi_1' = \frac{-f_3 f_3' \Delta_{1,2}}{\Delta_{1,2} \Delta_{2,3} + f_2 f_2'};$$

$$f' = \frac{f_1' f_2' f_3'}{\Delta_{1,2} \Delta_{2,3} + f_2 f_2'}.$$

Here f_k and f_k' mean the focal distances of the individual lenses, f' that of the whole lens system; $\Delta_{1,2}$ and $\Delta_{2,3}$ signify the optical distance (interval between the opposite focal points) between the first and second or between the second and third lens, and ξ_1 and ξ_1' indicate the distances of the entire focal point system from the front focal point of the first lens or from the back focal point of the third lens. All of these magnitudes are counted as positive in the beam direction.

We indicate the focal distances of the thin stray field lenses at the beginning and at the end of the field only with f_A and f_E, respectively; in addition we designate those of the homogeneous substitution field with f and of the real total field with F; these focal distances are to be taken as positive for a convex lens and negative for a concave lens. We moreover designate the distances between the focal points of the homogeneous substitution field with g' and g" and those of the real field system from the edge of the substitution field with G' and G"; these magnitudes are positive if the focal points lie outside the substitution field. The relationships of these magnitudes with those introduced above is as follows:

$$f_1' = -f_1 = f_A;$$
$$f_2' = -f_2 = f;$$
$$f_3' = -f_3 = f_E;$$
$$f' = F;$$
$$\Delta_{1,2} = -(f_A + g');$$
$$\Delta_{2,3} = -(f_E + g'');$$
$$G' = f_A - \xi_1.$$
$$G'' = f_E + \xi_1';$$

From this we get:

$$F = \frac{f f_A f_E}{(f_A + g')(f_E + g'') - f^2} \approx f\left(1 - \frac{g'}{f_A} - \frac{g''}{f_E}\right);$$

$$G' = \frac{g' f_A (f_E + g'') - f^2 f_A}{(f_A + g')(f_E + g'') - f^2} \approx g'\left(1 - \frac{g'}{f_A} - \frac{f^2}{g' f_E}\right);$$

$$G'' = \frac{g'' f_E (f_A + g') - f^2 f_E}{(f_A + g')(f_E + g'') - f^2} \approx g''\left(1 - \frac{g''}{f_E} - \frac{f^2}{g'' f_A}\right).$$

(10)

Since f_A and f_E are usually much larger than f, corresponding approximations were provided in equation (10). With the determination of the cardinal points of the real field system, the electron optical problem of the first order for transgradient electrical fields is completely solved.

5. Applications and Examples

The previous computations contain two presumptions which are not usually fulfilled completely in practical applications: first the rays are supposed to pass the stray field near the x-axis and second parallel to it. However, as we shall see from the following examples, the lens action of the stray fields is so slight that the computations can be used for all practically important cases without worry. Care must only be taken, by means of suitable auxiliary stops, to keep the beam from approaching the edges of the stops and condenser plates too closely.

A. Cylinder Condenser. This is where the two above presumptions are best fulfilled; however, the field is not homogeneous inwardly, while the field intensity is inversely proportional to the radius. But since the distance between the plates is usually smaller than the mean radius, the field for these purposes can be approximately considered as homogeneous.

Given a radial electrical field with a 10° aperture angle and plate radii R_1 = 5.2 cm, R_2 = 5.0 cm. Voltage is symmetrically grounded, thus $\phi_2 = -\phi_1$. Let the field be defined on each side by thin stops of 2b = 0.28 cm distance and d = 0.2 cm interval. First we shall look for the angle of deflection of those rays which move in the field in a circle of mean radius a = 5.1 cm; the path of these particles outside the field coincides with the optical axis.

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In addition the position of the focal points and of the focal distances of the entire field are computed.

We must first determine the length of the substitution field. Since k = = 0.1 cm, d/k = 2 and b/k = 1.4; we derive ξ/k = 0.8 for thin stops from Figure 3; the path curvature for this distance is the same as in the field. With a mean path radius a_e = 5.1 cm, the beam in each stray field will therefore be deflected by 54'. The total beam deflection is therefore ϕ_e = 54' + + 10° + 54' = 11°48'.

In order to compute the lens action, let us take from the previously cited work:

$$f_e = \frac{a_e}{\sqrt{2}\sin\sqrt{2}\,\Phi_e} = 12.5\text{ cm };$$

$$g_e = \frac{a_e}{\sqrt{2}\,\text{tg}\,\sqrt{2}\,\Phi_e} = 12.0\text{ cm}.$$

We also need the transmission ratio $\ddot{u}$ of the condenser; by this we mean the quotients of the accelerating current and the deflection current for those particles which go through the middle circular path. From the power equilibrium we get:

$$\ddot{u} = \frac{V}{\varphi_1 - \varphi_2} = \frac{1}{2 \cdot \ln\dfrac{R_1}{R_2}} \cdot \tag{11}$$

In our case we get: $\ddot{u}$ = 12.75. Now from Figure 4 we take $f_1/k \; (\phi_1 - \phi_2)^2 =$ = - 1.8; and thus $f_1 = f_A = f_E = -1.8 \cdot k \cdot 16 \cdot \ddot{u}^2$ = -470 cm.

From the approximation equation (10) we finally find F = 13.2 cm and G = = 12.7 cm. It can be seen from this that the lens activity of the stray fields is so slight that it does not essentially change that of the substitution field. Therefore the lens activity of the stray fields can usually be disregarded in considering the grazing section of the pencil of rays. However, disregarding

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deflecting action of the stray fields would have produced an error of about 18% in the angle of deflection and the focal distance.

B. Plane Plate Condenser. Given a condenser with a plate length of 1.2 cm and a plate interval of 2 k = 0.4 cm. Let the field be defined by a thick entrance stop of 2b = 0.2 cm distance and d = 0.4 cm interval and a thin exit stop of 2b = 0.4 cm distance and d = 0.1 cm interval. Let the plate inside the path be grounded (ϕ_1 = 0). To find the "sensitivity" of this arrangement, thus for example, the angle by which those beams which transmitted the current V = 10 ϕ_2 are deflected.

For the entrance stop d/k = 2, and b/k = 0.5; from Figure 3 we find: ξ/k = 0.7 and thus ξ = 0.14 cm. For the exit stop d/k = 0.5 and b/k = 1; analogously we find: ξ/k = 0.3 and therefore ξ = 0.06 cm. Since the ground can be presumed here to be unsymmetrical, we also have to consider the member Δξ/k = - 0.11 (ϕ_1 + ϕ_2)/V = - 0.01; however, in comparison with the length of the substitution field, its extent is very small (on each side - 0.002 cm) and may therefore be disregarded. Thus the length of the substitution field is L = 0.14 + 1.2 + 0.06 = 1.4 cm. We take the formula for computing the deflection from the work cited at the beginning. Here L·s = k and therefore s = = 0.143. In addition

$$F_\eta = \frac{\varphi_2}{2k} = \frac{0.1 \cdot V}{0.4} = \frac{V}{4}$$

and

$$E = \epsilon \cdot V.$$

From equation {4} there we thus find ρ = 5.425 and from equation {16}...Φ = = 10°21'. Without regard to the stray fields the angle of deflection would amount only to 8°32'; thus the error here also amounts to almost 20%. The exit position of the beam is located at $L\eta_2$ = L/2ρ = 0.13 cm from the axis; the distance from the edge of the condenser plate or stop amounts to 0.07 cm and is therefore still large enough for the above computations to be applied without worry.

The focal distances of the stray fields are computed here by means of equation (7) and Figures 4 and 5. We find f_E = -2,490 cm and f_A = -2,050 cm; thus the lens action of the stray fields here too is also so slight that it can be disregarded in relation to that of the substitution field (f = 42 cm).

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